Decomposition 2: Detailed decomposition

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Review & issues

- Oaxaca-Blinder decomposition:
  $\widehat{\Delta}_O^\mu = \overline{X}_B \left(\widehat{\beta}_B^\mu - \widehat{\beta}_A^\mu\right) + \left(\overline{X}_B - \overline{X}_A\right) \widehat{\beta}_A^\mu = \widehat{\Delta}_S^\mu + \widehat{\Delta}_X^\mu$
  =wage structure effect+composition effect

  - Choice of reference group 1) A solution: The idea is no longer to take any one group as reference but to assign each group an arbitrary weight. $\widehat{\Delta}_O^\mu = \left(\overline{X}_B - \overline{X}_A\right) \widehat{\beta} + \overline{X}_A(\widehat{\beta} - \widehat{\beta}_A) + \overline{X}_B(\widehat{\beta} - \widehat{\beta}_B), \widehat{\beta} = \lambda\widehat{\beta}_A + (1-\lambda)\widehat{\beta}_B, \lambda \in [0,1]$. A popular choice of λ is to use the share of the two groups in the population. (Cahuc, Carcillo, and Zylberberg, 2014, p. 510) (Table 3 in the NBER paper); 2) Which are the more appropriate reference group? The answer depends on complicated issues related to the degree to which the data we use can accurately measure differentials in personal and job characteristics. (O'Neill and O'Neill, 2006)

- Variance decomposition=within-group component+between-group component

- Juhn-Murphy-Pierce (1993)

- Machado and Mata (2005) and Melly (2005)

  - $Q_{g,\tau}(Y|X) = F^{-1}_{Y_g|X_g}(\tau, X) = X\beta_{g,\tau}$ and $Y^C_{As} = X'_{A,\tau s}\beta_{B,\tau s}$
  - $Q_\tau(y) \neq E[Q_\tau(Y|X)]$ The expectation of the conditional median does not produce the median of the marginal distribution. In addressing this problem, MM (2005) propose a simulation-based technique and Melly (2005) proposes integrating the entire conditional distribution function by integrating over the full set of covariates. (Detailed decompositions are not allowed in MM (2005)/Melly (2005))
  - Difference in contribution of residuals 1) To the extent that residual wage dispersion is due to unmeasured differences in human capital investment, the residual dispersion should increase when the 'return' to human capital increases. For instance, when the return to years of schooling increases, we

can expect that the return to (unmeasured) school quality would also increase. (Lemieux, 2002); 2) The changes in covariates do not only affect the level of wages but also increase within-group inequality. Methods that do not account for the dependence between residuals and characteristics overstate the effects of residuals and understate the effects of characteristics. (Melly, 2005)

- Reweighting methods: DiNardo, Fortin, and Lemieux (1996)
  - Limitation of reweighting approach?
- Chernozhukov, Fernandez-Val, and Melly (2009) suggest to estimate directly distributional regression models for $F_{Y|X}(\cdot,\cdot)$.

RIF-regression methods: Firpo, Lemieux and Fortin (2009)

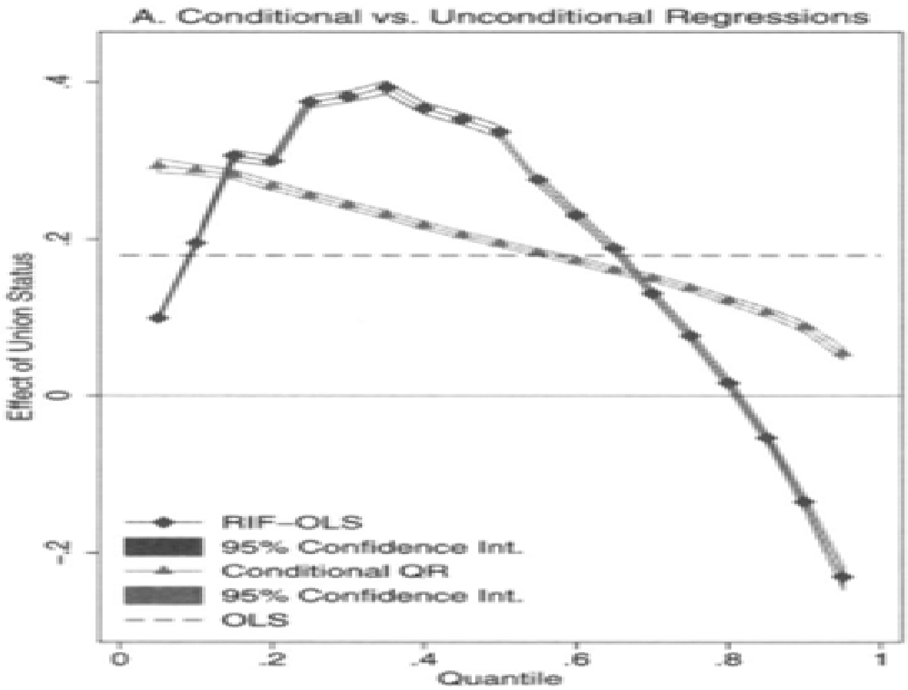
- Empirical researchers are often interested in changes in the quantiles, denoted Q_τ, of the marginal (unconditional) distribution, $F_Y(y)$. ex) The direct effect $dQ_\tau(p)/dp$ of increasing the proportion of unionized workers, $p = Pr[X = 1]$, on the τth quantile of the distribution of wages ($X = 1$: unionized).

Conditional vs. Unconditional

- The coefficient β_τ from a single conditional quantile regression, $\beta_\tau = F_Y^{-1}(\tau|X = 1) - F_Y^{-1}(\tau|X = 0)$, is different from $dQ_\tau(p)/dp$. (The law of iterated expectations does not apply in the case of quantiles, so $Q_\tau(y) \neq E[Q_\tau(Y|X)]$.)
- The influence function $IF(Y;\nu,F_Y)$ of a distributional statistic $\nu(F_Y)$ represents the influence of an individual observation on that distributional statistic.
- For the τth quantile, RIF is following:

$$RIF(y;Q_\tau) = Q_\tau + IF(y;Q_\tau) = Q_\tau + \frac{\tau - \mathbb{1}\{y \leq Q_\tau\}}{f_Y(Q_\tau)}$$

- One convenient feature of the RIF is that its expectation is equal to $\nu(F_Y)$. In the case of quantiles, $E[RIF(Y;\nu,F_Y)|X] = m_\tau(X)$ (or $Q_\tau(y) = E[RIF(Y|X)]$).



- The unconditional union effect is highly nonmonotonic; the effect first increases from about 0.1 at the 5th quantile to about 0.4 about the 35th quantile, before declining and eventually reaching a large negative effect of over -0.2 at the 95th quantile (FFL, 2009).

- By contrast, standard (conditional) quantile regression estimates decline almost linearly which simply means that unions reduce within-group dispersion (FFL, 2009).

- One more! The first bracketed component represents the wage structure component for the wage decomposition methodology literature and identifies the average treatment effects of the treated (ATT) in the programme evaluation literature. (Moving from group A to group B is interpreted to be "the treatment".)

$$\begin{aligned}\widehat{\Delta}_O^\mu &= E[Y_B] - E[Y_A] \\ &= \{E[Y_B|D_B = 1] - E[Y_A|D_B = 1]\} \\ &+ \{E[Y_A|D_B = 1] - E[Y_A|D_A = 1]\}\end{aligned}$$

RIF-regressions & OB decomposition

- A RIF-regression is similar to a standard regression, except that the dependent variable, Y, is replaced by the RIF of the statistic of interest.
- Coefficients of the unconditional quantile regression for each group:

$$\widehat{\gamma}_{g,\tau} = \left(\sum X_i \cdot X_i^T\right)^{-1} \cdot \sum \widehat{RIF}(Y_{gi}; Q_{g,\tau}) \cdot X_i$$

- OB decomposition for any unconditional quantile:

$$\begin{aligned}\widehat{\Delta}_O^\tau =& \overline{X}_B \left(\widehat{\gamma}_{B,\tau} - \widehat{\gamma}_{A,\tau}\right) + \left(\overline{X}_B - \overline{X}_A\right) \widehat{\gamma}_{A,\tau} \\ =& \widehat{\Delta}_S^\tau + \widehat{\Delta}_X^\tau \\ =& \widehat{\Delta}_S^\tau + \sum (\overline{X}_{Bk} - \overline{X}_{Ak}) \widehat{\gamma}_{Ak,\tau}\end{aligned}$$

FFL (2009) and CFVM (2009)

- In FFL (2009) and CFVM (2009), models are estimated for explaining the determinants of the proportion of workers earning less than a certain wage. In CFVM (2009) estimates for proportions are *globally* inverted back into the quantiles, while while in FFL (2009) the inversion is only performed *locally*.

References

[1] Firpo, S., N. Fortin, and T. Lemieux, 2009. Unconditional Quantile Regressions. *Econometrica* 77 (3), 953–973.

[2] Firpo, S., T. Lemieux, and N. Fortin, 2010. Decomposition Methods in Economics. NBER Working Paper 16045.

[3] Lemieux, T., 2002. Decomposing Changes in Wage Distributions: A Unified Approach. *Canadian Journal of Economics* 35 (4), 646–688.

[4] Melly, B., 2005. Decomposition of differences in distribution using quantile regression. *Labour Economics* 12, 577–590.

[5] O'Neill, J., and O'Neill, D. 2006. What do wage differentials tell us about labor market discrimination? *Research in Labor Economics* 24, 293–357.

Figure 3: RIF Regressions: Inverting Locally

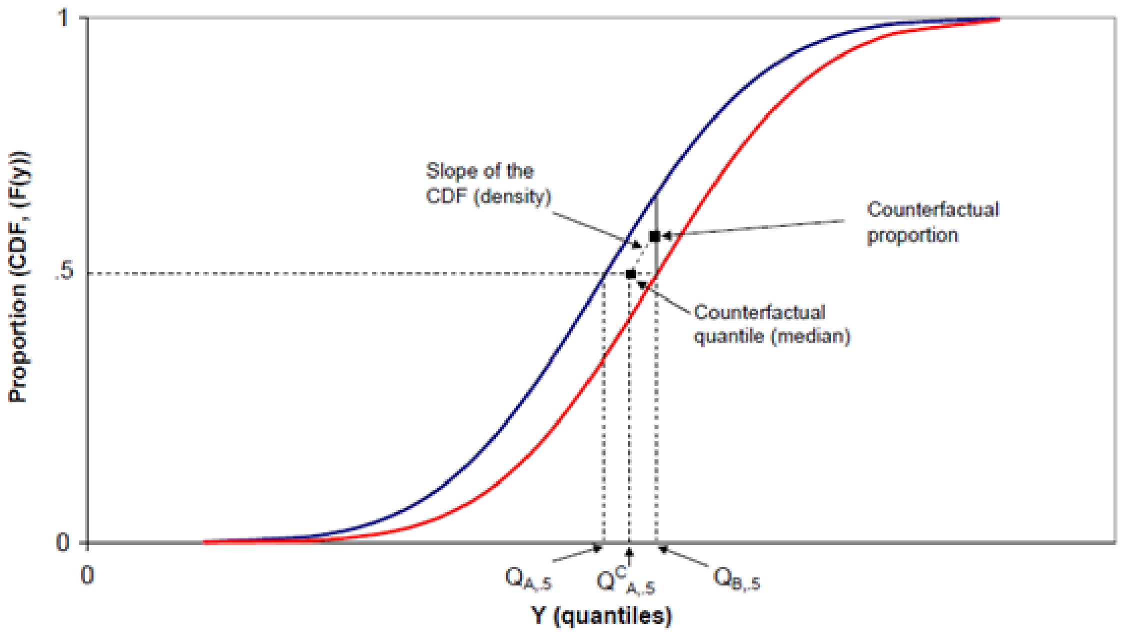


Figure 2: Inverting Globally